

Calculation policy: The LETTA Trust

Purpose of a calculation policy

- About ensuring a consistent approach across the school, with each year group building upon the strategies and layout used in previous year groups.
- Enabling the school to use consistent resources, pictorial representations and language when doing calculations, enabling children to have a reduced cognitive load as they are building on what they have already learnt













Key parts of our approach:

Concrete, Pictorial, Abstract approach

- We do not believe that children 'graduate' from concrete, to pictorial and finally to abstract; instead, children should be exposed to the pictorial and concrete alongside the abstract strategy they are learning.
- 'From concrete manipulatives and experiences, students are *guided to uncover abstract mathematical concepts or results...* The role of the teacher is that of *facilitator*, who guides students through the concrete, pictorial and abstract levels of understanding by providing *appropriate scaffolding and feedback*.' - Ministry of Education. 2012
- The focus is on ensuring children have an understanding of the mathematics the sits behind the strategy, rather than on finding answers - Askew, M. 2012

Therefore:

- **In lessons:** teachers should model solving using the concrete and pictorial alongside the abstract calculation - this enables children to see the underlying concepts. *Using the visualiser here really helps.*
- **In books:** children should be expected to draw the pictorials (or use already-printed ones first) alongside solving the abstract until they are confident in the maths (this means having the ability to explain their understanding, not just getting answers correct).


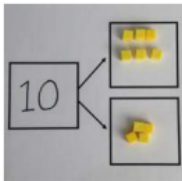

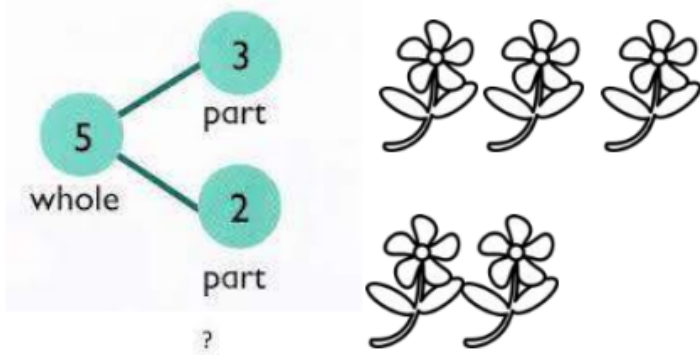
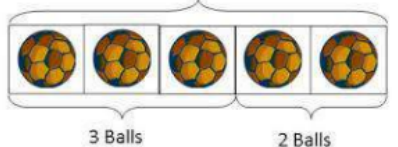
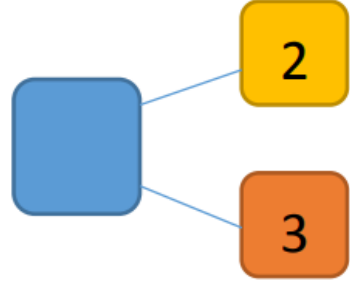


Solve...		$26 + 33 =$					
Model		Calculations					
<table border="1"><thead><tr><th>Tens</th><th>Ones</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table>	Tens	Ones					$\begin{array}{r} 26 \\ + 33 \\ \hline 59 \end{array}$
Tens	Ones						
							
							
		$26 + 35 =$					


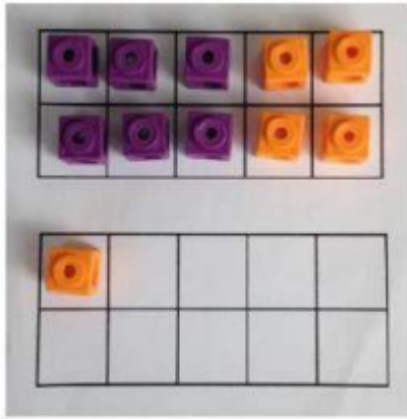
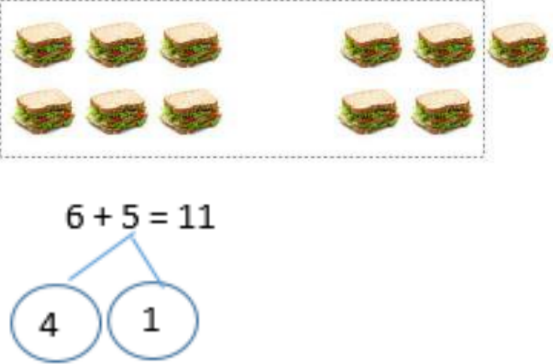
Key vocabulary: **exchange**
Can we exchange any ones?



Calculation guidance: Addition


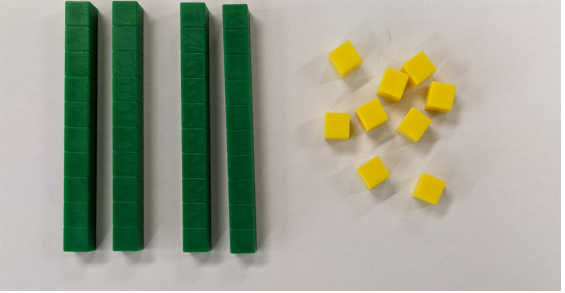
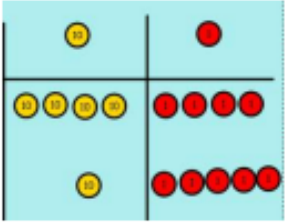
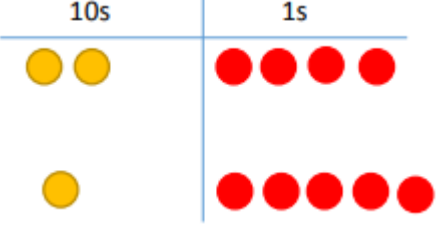
Year 1:

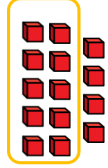
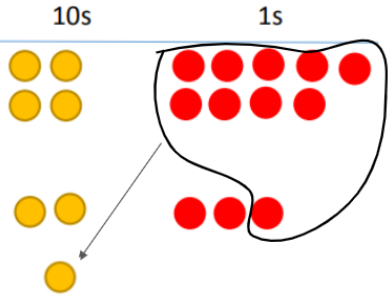
	concrete	pictorial	abstract
<p>To represent and use number bonds and related subtraction facts within (and including) 10</p> <ul style="list-style-type: none"> - Number bonds of 5, 6, 7, 8, 9, 10 	<p>Cubes</p> <ul style="list-style-type: none"> - Use to add two numbers together as a group or in a bar <p>$4 + 3 = 7$</p>  <p>$10 = 6 + 4$</p>  <p>$8 + 1 = 9$</p> 	<p>Part / whole method alongside images</p> <ul style="list-style-type: none"> - Create your parts and label - Add altogether to find the whole <p>$3 + 2 = 5$</p>  <p>Bar model</p> 	<p>Part / whole model alongside all number sentences to move into the abstract</p> <p>$2 + 3 = 5$</p> <p>$3 + 2 = 5$</p> <p>$5 = 3 + 2$</p> <p>$5 = 2 + 3$</p> 
<p>Counting: add 1 and 2 digit numbers within 20 including zero:</p>	<p>Bead string</p> <ul style="list-style-type: none"> - Start with the larger number and then count on the next number 1 by 1 <p>$12 + 5 = 17$</p> 	<p>Number line</p> <ul style="list-style-type: none"> - Start at larger number - Count on in ones 	<p>$5 + 3 = 8$</p>

	concrete	pictorial	abstract
<p>Regrouping to make 10: add 1 and 2 digit numbers within 20 including zero</p>	<p>Bead string</p> <ul style="list-style-type: none"> - Make larger number first - Use the smaller number to make 10 - Add the leftover amount <p>$9 + 3 = 12$</p>  <p>Cubes</p> <ul style="list-style-type: none"> - Make larger number on 10 frame - Use the smaller number to make 10 - Add leftover cubes in next 10 frame <p>$6 + 5 = 11$</p> <ul style="list-style-type: none"> - $6 + 4 = 10$ - $10 + 1 = 11$ 	<p>Pictorial</p> <ul style="list-style-type: none"> - Group 10 - Count on what is left <p>$6 + 5 = 11$</p> <ul style="list-style-type: none"> - $6 + 4 = 10$ - $10 + 1 = 11$  <p>$6 + 4 = 10$</p> <p>$10 + 1 = 11$</p>	<p>$6 + 5 = 11$</p>

Year 2:

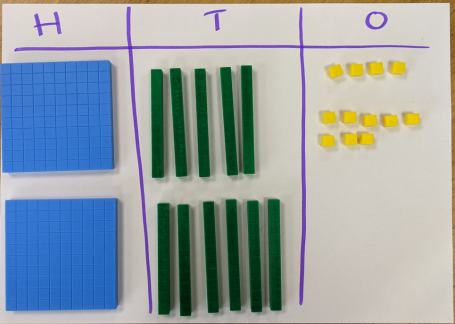
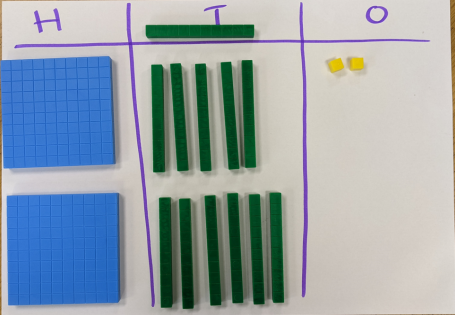
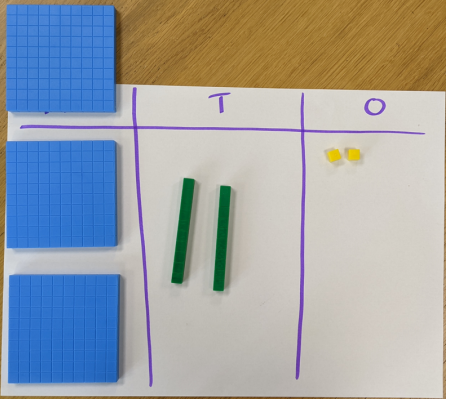
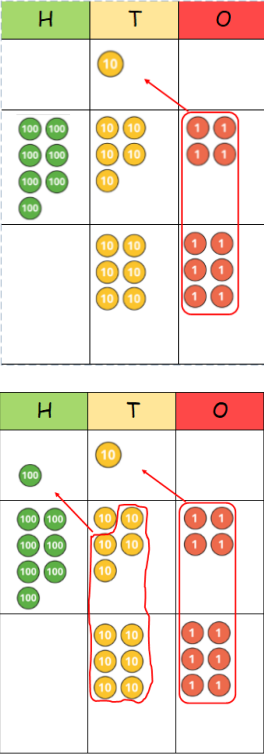
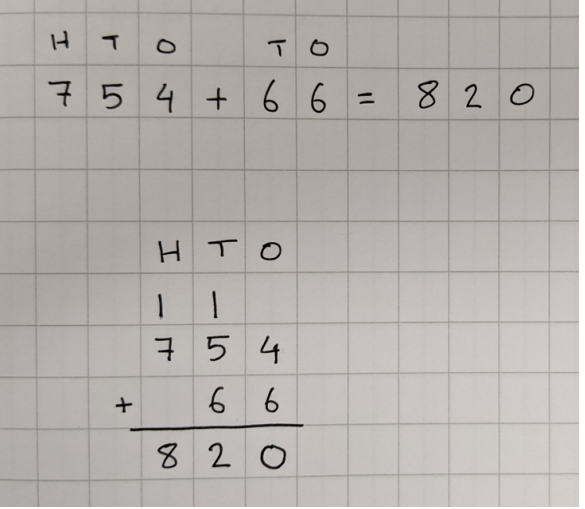
To also understand: that addition is commutative

	concrete	pictorial	abstract
Column method without regrouping	<p>Base 10 $17 + 32 =$ Begin to introduce the formal column method layout by placing the numbers one on top of the other Create your two numbers using base 10</p>  <p>Add (group) the ones together - what do you have? Add (group) the 10s together - what do you have? What is the total?</p>  <p>Counters</p> <ul style="list-style-type: none"> - Create your numbers - Add (group) the ones together - how many do you have? - Add (group) the tens together - how many do you have? <p>$44 + 15 = 59$</p> 	<p>After physically using the base 10 and counters, children can use drawings to build their understanding</p>  <p>Again, count the ones first - how many do you have?</p>	<p>Formal written method here</p>

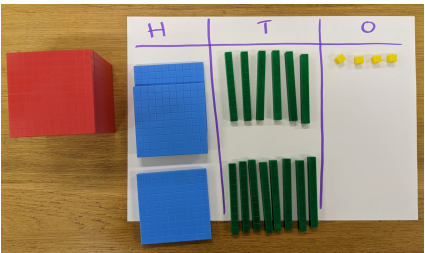
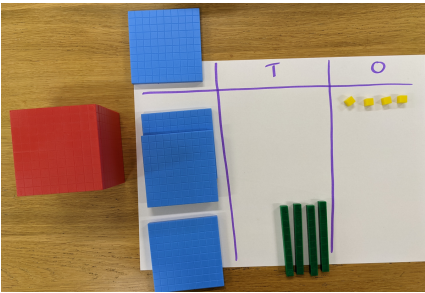
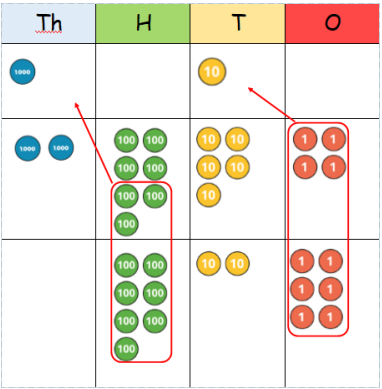
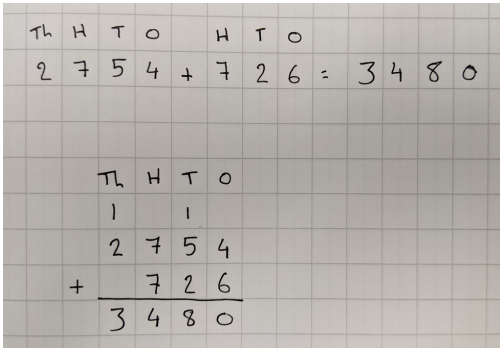
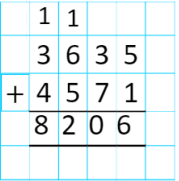
	concrete	pictorial	abstract																																				
Column method with regrouping	<p>Base 10</p> <ul style="list-style-type: none"> - <i>N.b. children will need to understand 10 ones is the same as 1 ten</i>  <p>e.g. 14 ones is 1 ten and 4 ones</p> <p>$49 + 23 = 72$</p> <ul style="list-style-type: none"> - Create both numbers on a place value grid <table border="1" data-bbox="655 695 893 905"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> </tbody> </table> <ul style="list-style-type: none"> - Add up the ones - when you have 10, exchange for 1 ten and place in 10s column - What is left? - Add up the 10s - What is your total? <table border="1" data-bbox="655 1136 893 1423"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> <tr><td> </td><td></td></tr> </tbody> </table>	10s	1s																	10s	1s																	<p>Drawing counters or base 10 to support understanding</p> <p>$49 + 23 = 72$</p> 	<p>$40 + 9$</p> <p><u>$20 + 3$</u></p> <p>$60 + 12 = 72$</p>
10s	1s																																						
10s	1s																																						
Adding three 1 digit numbers																																							

Year 3:

	concrete	pictorial	abstract
Using number line to cross the 10s / 100s barriers		<p>Adding 1s 418 + 7</p> <p>$418 + 7 = 425$</p> <p>$420 + 5 = 425$</p> <p>Adding 10s 583 + 50</p> <p>$583 + 50 = 633$</p>	

	concrete	pictorial	abstract
<p>To add numbers with up to 3 digits using formal column method (including regrouping). Using inverse to check answers</p>	<p>Base 10 254 + 68 Make both numbers on a place value grid</p>  <p>Add the ones digits - exchange 10 ones for one ten</p>  <p>Add the tens digits - exchange 10 tens for one hundred</p>  <p><i>Image using counters</i></p>	<p>754 + 66</p> 	

Year 4:

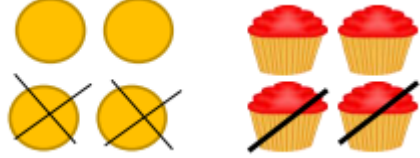
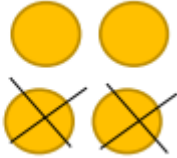

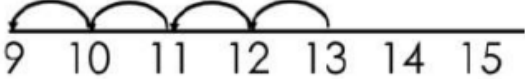
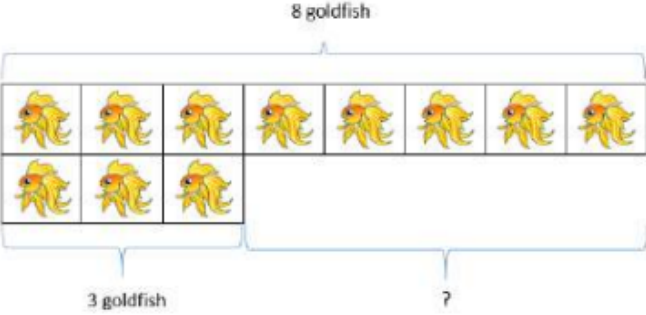
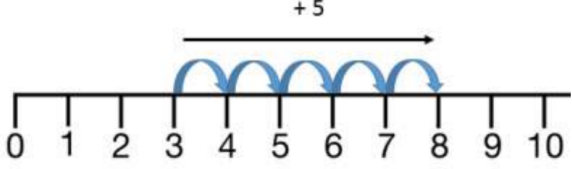
	concrete	pictorial	abstract
Be able to add ones, tens, hundreds and thousands using place value knowledge	<p>Photo using dienes cubes</p> <p>Image using counters</p>		<p>Examples:</p> $3,400 + 50 = 3,450$ $2,073 + 20 = 2,093$ <p>Children to understand that these questions do not require using the formal written column method</p>
Use column method to add numbers with up to 4 digits (exchanging and regrouping). Use the inverse to check answers	<p>Base 10 1264 + 180 Make both numbers on a place value grid</p>  <p>Add each column together, starting with the ones. Group 10 and exchange for one of the next column</p>  <p>Can also be made using counters</p>	<p>2,754 + 726 = 3,480</p> 	 <p>Example with sentence stems:</p> $3,635 + 4,571 = 8,206$ <p>I have <u>6</u> ones, so I <u>do not</u> need to make an exchange.</p> <p>I have <u>10</u> tens, so I <u>do</u> need to make an exchange.</p> <p>I have <u>12</u> hundreds, so I <u>do</u> need to make an exchange.</p> <p>I have <u>8</u> thousands, so I <u>do not</u> need to make an exchange.</p> 

Year 5 & Year 6:

	concrete	pictorial	abstract
To add numbers with more than 4 digits	<i>Embed from Y4 up to 4 digits. Move to pictorial & abstract for greater than 4 digits</i>	<i>Embed from Y4</i>	

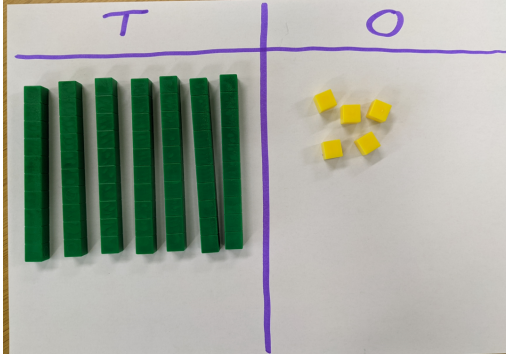
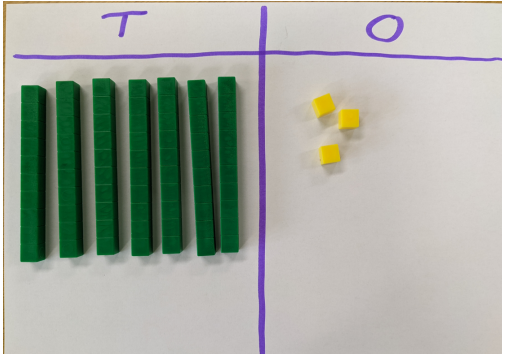
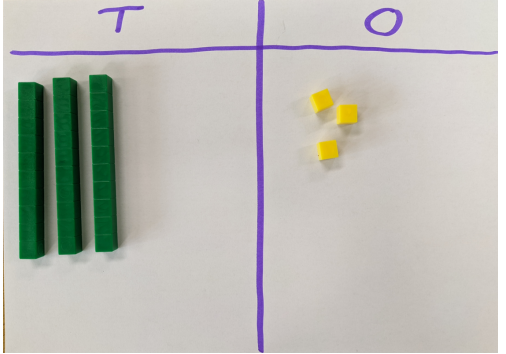
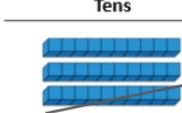

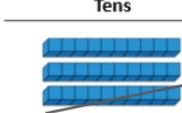

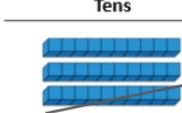

Calculation guidance: Subtraction

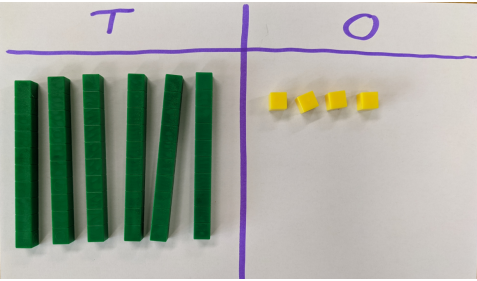
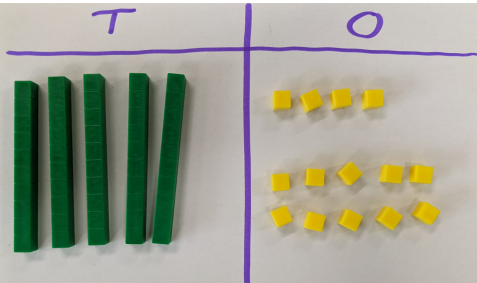
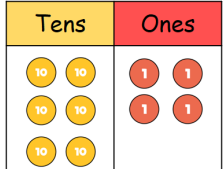
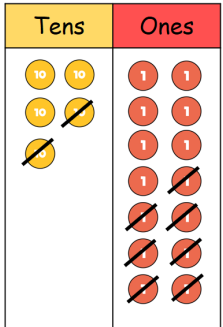
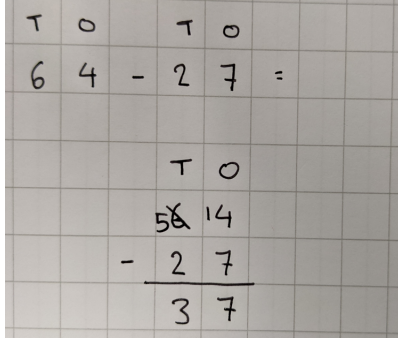
Year 1:

	concrete	pictorial	abstract
Taking away ones	<p>Counters / objects</p> <ul style="list-style-type: none"> - Use to show how objects can be taken away <p>$4 - 2 = 2$</p> 	<p>Drawn counters / objects</p> <ul style="list-style-type: none"> - Cross out to show what has been taken away <p>$4 - 2 = 2$</p> 	<p>$4 - 2 = 2$</p>
Counting back	<p>Bead string</p> <ul style="list-style-type: none"> - Make the starting number (minuend). Move the beads along the bead string as you count back in ones  <p>$13 - 4 = 9$</p>	<p>Number line / number track</p> <ul style="list-style-type: none"> - Start at the minuend and count back in ones, showing the jumps on the number line 	<p>Put 13 in your head Count back 4 What number are you at?</p> <p><i>Use fingers to help</i></p>
Find the difference	<p>Cubes</p> <ul style="list-style-type: none"> - Use cubes to make towers or bars to find the difference 	<p>Number line</p> <ul style="list-style-type: none"> - Find the two numbers you are finding the difference of - Count on from the smaller number to the bigger number  <p>Bar model</p> <ul style="list-style-type: none"> - Draw simple comparative bar model to find the difference between 2 numbers <p><i>example</i></p>	

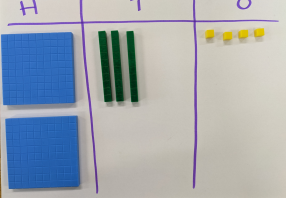

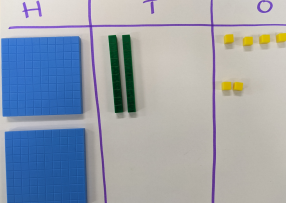
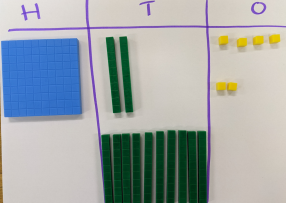
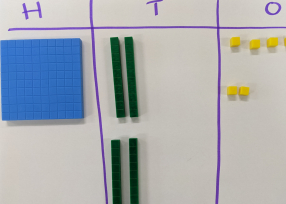
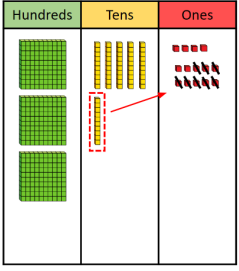
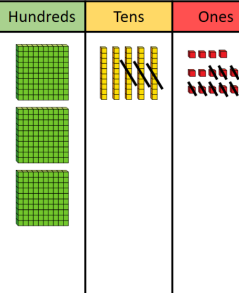
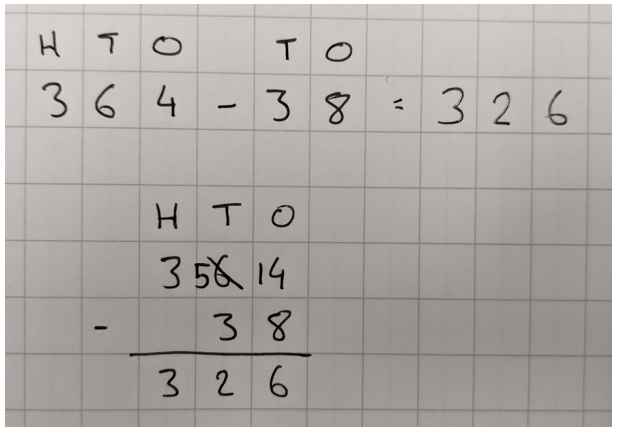
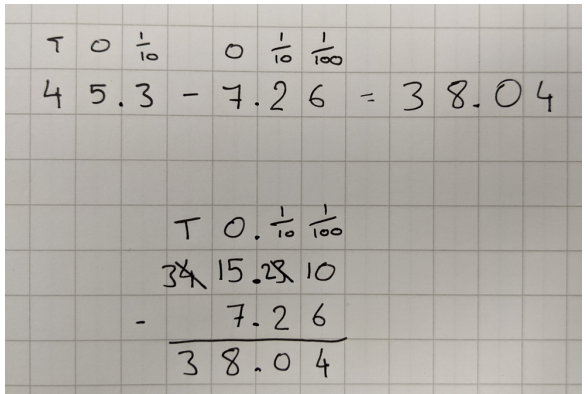
Year 2:

To also understand: subtraction is **not** commutative

	concrete	pictorial	abstract									
Column subtraction without exchange	<p>Base 10 75 - 42 = 33 Make the starting number (minuend) in place value columns</p>  <p>Take the smaller number (subtrahend) away, starting with the ones column</p>   <p>Can also be made using place value counters</p>	<p>Continue using base 10 models - children can eventually draw their own</p> <p>34 - 12</p> <table border="1"><thead><tr><th colspan="2">Model</th><th>Calculations</th></tr></thead><tbody><tr><td>Tens</td><td>Ones</td><td>$\begin{array}{r} 34 \\ - 12 \\ \hline 22 \end{array}$</td></tr><tr><td></td><td></td><td></td></tr></tbody></table>	Model		Calculations	Tens	Ones	$\begin{array}{r} 34 \\ - 12 \\ \hline 22 \end{array}$				<p>To be shown alongside the model until children are confident in the maths going on</p>
Model		Calculations										
Tens	Ones	$\begin{array}{r} 34 \\ - 12 \\ \hline 22 \end{array}$										
												

	concrete	pictorial	abstract
<p>Column subtraction with exchange</p>	<p>Base 10 64 - 27 Create the starting number (minuend) in place value columns</p>  <p>Begin subtraction in the ones column. I can't subtract 7 from 4 so I need to exchange 1 ten for 10 ones.</p>  <p>I can then do 14 - 7 which is 7</p>  <p>I can then move to the tens column. 50 - 20 = 30</p> 	<p>Children to have drawn counters at first, then begin drawing their own - can also be made using base 10</p> <p>64 - 27 = 37 - Create the starting number</p>  <p>- 4 - 7 is not possible here, so we exchange 1 ten for 10 ones. We now can do 14 - 7 = 7</p>  <p>- We can then do 50 - 20 = 30 (3 tens)</p> 	<p>To be shown alongside the pictorial until children are confident</p> 

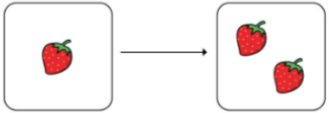




Year 3 upwards:

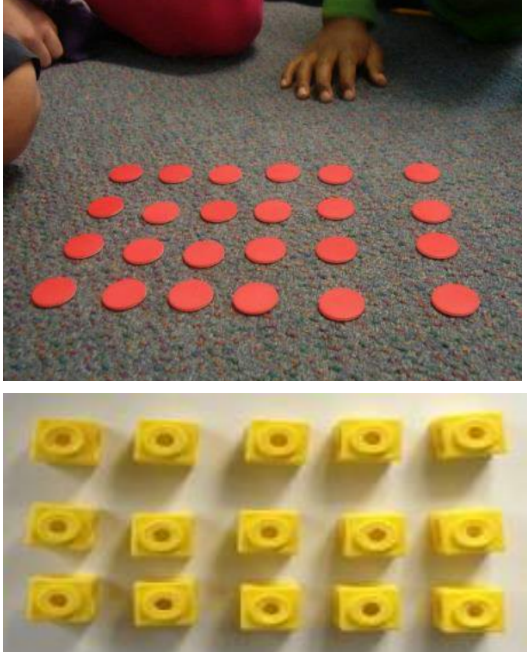
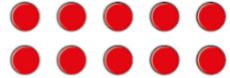
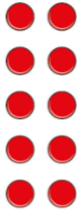
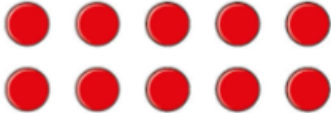
	concrete	pictorial	abstract																								
<p>To subtract numbers with up to 3 digits using formal method of column subtraction (exchanging and regrouping). Using inverse to check answers</p>	<p>Base 10 on place value grid - n.b. The example below is the final step in understanding in Y3</p> <p>234 - 88 = 156</p> <p>Make starting number on place value grid</p>  <p>Start with the ones. Can I subtract 8 from 4? No - I need to exchange 1 ten for 10 ones - I now have 14 ones</p>  <p>Now I can subtract my ones $14 - 8 = 6$</p>  <p>I can now look at my tens column. $30 - 80$. I can't do that - I need to exchange 1 hundred for 10 tens.</p>  <p>I now have $120 - 80 = 40$</p>  <p>I am not taking any hundreds away from my hundreds column so my answer is 146</p>	<p>Base 10 and then move on to counters</p> <p>364 - 38 = 326</p> <ul style="list-style-type: none"> - Make starting number (minuend) - Begin by subtracting the ones. If you cannot do this, exchange 1 ten for 10 ones  <table border="1" data-bbox="1617 514 1751 661"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>3</td><td>6 14</td><td></td></tr> <tr><td></td><td>3</td><td>8</td></tr> <tr><td></td><td></td><td></td></tr> </table> <ul style="list-style-type: none"> - Then, move onto the tens column. Again, if you cannot subtract then exchange 1 hundred for 10 tens  <table border="1" data-bbox="1617 913 1795 1081"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>3</td><td>6 14</td><td></td></tr> <tr><td></td><td>3</td><td>8</td></tr> <tr><td>3</td><td>2</td><td>6</td></tr> </table> <p>Children should get into the habit of checking their answers using the inverse (addition) operation</p>	H	T	O	3	6 14			3	8				H	T	O	3	6 14			3	8	3	2	6	<p>Year 3 example:</p>  <p>note: when exchanging, the digit exchanged should be the same size (e.g. the 1 ten going into the ones column)</p> <p>Example with decimals:</p> 
H	T	O																									
3	6 14																										
	3	8																									
H	T	O																									
3	6 14																										
	3	8																									
3	2	6																									

Calculation guidance: Multiplication

Year 1:

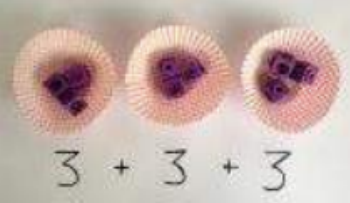


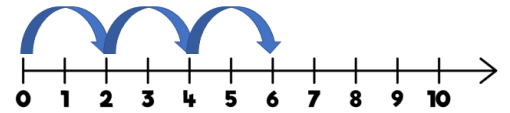
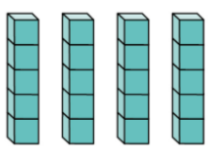
- Need to be able to count in 2, 5, 10
- Understand that multiplication involves equal groups

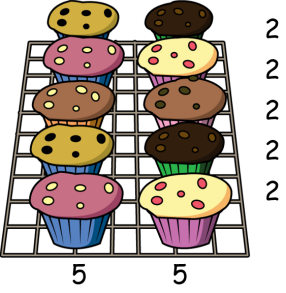
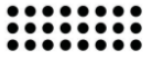
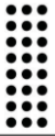
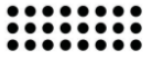
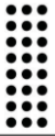
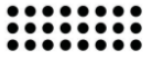
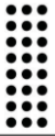
	concrete	pictorial	abstract
<p>To double numbers within 10 (then within 20)</p>	<p>Children can use fingers (join hands together to show doubles)</p>	<p>Complete the sentences. Use the pictures to help you.</p> <p>a) </p> <p>Double 1 is <input style="width: 20px; height: 20px;" type="text"/></p> <p>1 + 1 = 2</p>	
<p>Recognise repeated addition and representing with multiplication equations (2, 5, 10 x tables)</p>	<p>Children should begin by building an understanding of counting in 2s, 5s and 10s</p> <p>Using objects to add equal groups</p> <p>7 lots of 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14 - How many? (counting up in twos)</p>  <p>3 groups of 5 = 5 + 5 + 5 = 15</p> 	<p>Children draw out objects / counters to show the repeated addition</p> <p>There are 3 pots. There are 2 pencils in each pot</p> 	<p>Children should write the addition sentences to describe the objects and pictures</p>  <p>2 + 2 + 2 = 6</p>

	concrete	pictorial	abstract
Making arrays	<p>Counters or cubes</p>  <p>Sentences to support:</p> <p>There are <input type="text"/> counters in each row.</p> <p>There are <input type="text"/> rows.</p> <p>There are <input type="text"/> counters altogether.</p>	<p>Draw arrays in different rotations to find commutative sentences</p>  <p>Complete the sentences.</p> <p>a) There are <input type="text"/> counters in each row.</p> <p>There are <input type="text"/> rows.</p> <p>There are <input type="text"/> counters altogether.</p> 	<p>Use the arrays to write multiplication sentences and reinforce repeated addition</p>  <p>$5 + 5 = 10$ $5 \times 2 = 10$</p> <p>$2 + 2 + 2 + 2 + 2 = 10$ $2 \times 5 = 10$</p>

Year 2:

By the end of Y2, children should be secure in **2, 5, 10** times tables

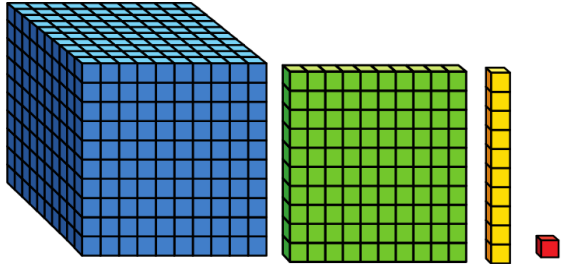
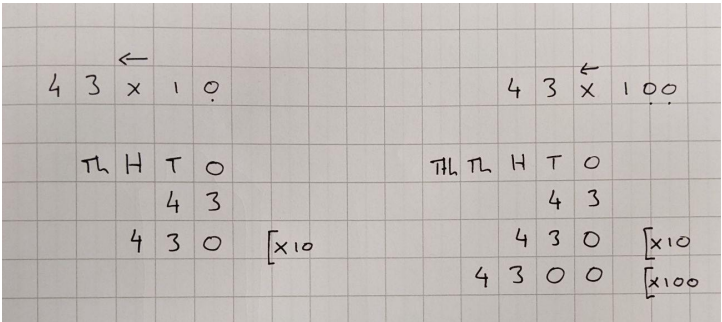
	concrete	pictorial	abstract										
<p>Recognise repeated addition and representing with multiplication equations (2, 5, 10 x tables)</p> <p>- Equal groups</p>	<p>Children should begin by building an understanding of counting in 2s, 5s and 10s</p> <p>Using objects to add equal groups</p> <p>3 groups of 3 = 3 + 3 + 3 = 9</p>  <p>3 groups of 5 = 5 + 5 + 5 = 15</p> 	<p>Children draw out objects / counters to show the repeated addition - illustrate on a numberline</p>  <p>There are <input type="text" value="3"/> equal groups of <input type="text" value="2"/></p> <p><input type="text" value="2"/> + <input type="text" value="2"/> + <input type="text" value="2"/> = <input type="text" value="6"/></p>  <p>Once embedded, begin to show the multiplication sentence alongside</p>  <p>There are <input type="text"/> equal groups with <input type="text"/> in each group.</p> <p><input type="text"/> + <input type="text"/> + <input type="text"/> + <input type="text"/> = 20</p> <p><input type="text"/> x <input type="text"/> = 20</p>	<p>When moving away from pictorial, ensure children continue to see the relationship between repeated addition and multiplication sentence:</p> <p>Complete the table.</p> <p>The first one has been done for you.</p> <table border="1"> <thead> <tr> <th>Addition</th> <th>Multiplication</th> </tr> </thead> <tbody> <tr> <td>2 + 2 + 2 + 2</td> <td>4 x 2</td> </tr> <tr> <td>5 + 5 + 5</td> <td></td> </tr> <tr> <td>3 + 3 + 3 + 3 + 3</td> <td></td> </tr> <tr> <td></td> <td>2 x 10</td> </tr> </tbody> </table>	Addition	Multiplication	2 + 2 + 2 + 2	4 x 2	5 + 5 + 5		3 + 3 + 3 + 3 + 3			2 x 10
Addition	Multiplication												
2 + 2 + 2 + 2	4 x 2												
5 + 5 + 5													
3 + 3 + 3 + 3 + 3													
	2 x 10												

	concrete	pictorial	abstract									
<p>Arrays: Understand multiplication as number of groups with total unknown - also understand multiplication is commutative</p>	<p>Create arrays using physical objects / cubes</p> <ul style="list-style-type: none"> - How many rows? - How many in each row? - How many in cakes? 	<p>Continue concrete work with pictorials and counters. Build understanding of multiplication being commutative</p> <table border="1" data-bbox="1359 394 2047 699"> <thead> <tr> <th>Multiplication</th> <th>Array 1</th> <th>Array 2</th> </tr> </thead> <tbody> <tr> <td>3×8</td> <td></td> <td></td> </tr> <tr> <td>2×5</td> <td></td> <td></td> </tr> </tbody> </table>	Multiplication	Array 1	Array 2	3×8			2×5			<p>$5 + 5 = 10 \rightarrow 2 \times 5 = 10$ $2 + 2 + 2 + 2 + 2 = 10 \rightarrow 5 \times 2 = 10$</p>
Multiplication	Array 1	Array 2										
3×8												
2×5												

Year 3 & Y4:

By the end of **Y3**, children should be secure in 2, 5, 10, **3, 4, 8** times tables

By the end of **Y4**, children should be secure in **all times tables**

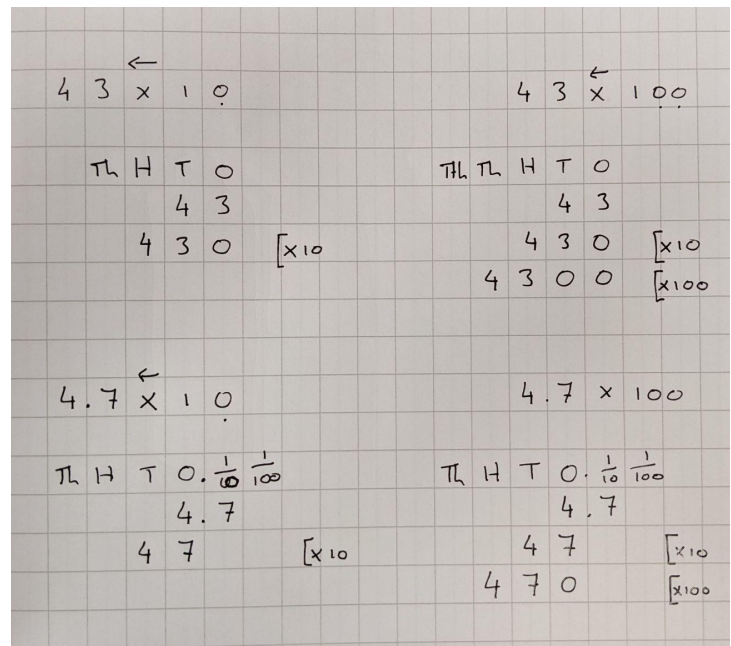
	concrete	pictorial	abstract																																																																																																												
<p>To recognise the effect of multiplying 1 digit numbers by 10 and 100</p>	<p>Children need to understand the relationship between ones, tens, hundreds and thousands. They can do this by seeing how many they need to exchange for the following power of 10.</p> <p>E.g.</p> <ul style="list-style-type: none"> - 10 ones is the same as 1 ten. Therefore 1 ten is 10 times the size of 1 one  <p>1 ten is 10 times the size of 1 one 1 hundred is 10 times the size of 1 ten 1 thousand is 10 times the size of 1 hundred</p>	<p>Children should understand the effect of multiplying by powers of 10 by first using repeated addition to multiply numbers 10 times.</p> <table border="1" data-bbox="1368 569 1635 869"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> <tr><td></td><td>10</td><td>10</td></tr> </tbody> </table> <p>Each row has <u>3</u> tens and <u>2</u> ones. Each row has <u>32</u> There are <u>10</u> rows. The calculation is <u> </u> × <u> </u> = <u> </u></p> <p>- I exchange 20 ones for 2 tens</p> <table border="1" data-bbox="1368 940 1635 1241"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> <tr><td></td><td>20</td><td></td></tr> </tbody> </table> <p>Each row has <u>3</u> tens and <u>2</u> ones. Each row has <u>32</u> There are <u>10</u> rows. The calculation is <u>32</u> × <u>10</u> = <u> </u></p> <p>- I exchange 30 tens for 3 hundreds</p> <table border="1" data-bbox="1368 1312 1635 1612"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr><td>300</td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> </tbody> </table> <p>Each row has <u>3</u> tens and <u>2</u> ones. Each row has <u>32</u> There are <u>10</u> rows. The calculation is <u>32</u> × <u>10</u> = <u> </u></p> <p>- I now have 320</p>	H	T	O		10	10		10	10		10	10		10	10		10	10		10	10		10	10		10	10		10	10		10	10		10	10	H	T	O		20			20			20			20			20			20			20			20			20			20			20		H	T	O	300																																	
H	T	O																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
	10	10																																																																																																													
H	T	O																																																																																																													
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
	20																																																																																																														
H	T	O																																																																																																													
300																																																																																																															

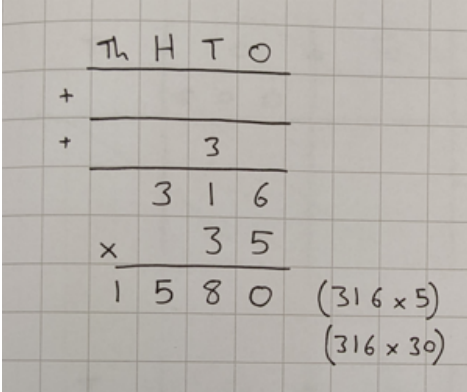
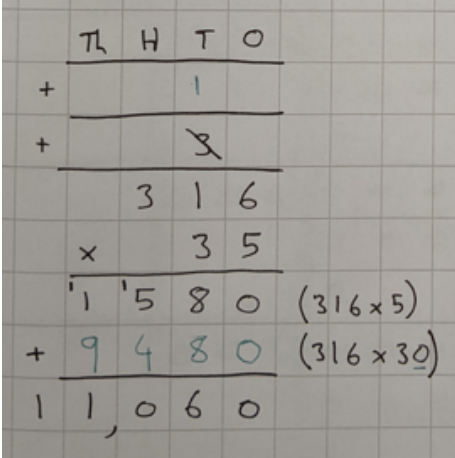
	concrete	pictorial	abstract
<p>Column multiplication - no exchange</p> <ul style="list-style-type: none"> - To multiply 2 digit numbers by 1 digit numbers using the formal written methods 	<p>32 x 3 = 96</p> <ul style="list-style-type: none"> - 32 x 3 means I am multiplying 32, three times <ul style="list-style-type: none"> - Start by groupings the ones. I have 6 ones. <ul style="list-style-type: none"> - Group tens. I have 9 tens, which is 90. - My answer is 96 	<p>Drawn counters</p> <p>32 x 3 = 96</p> <ul style="list-style-type: none"> - N.B. Layout - column at top for exchanges; not needed yet but good to embed ready for new learning - I have 3 lots of 32 - Begin in the ones column - I have 3 lots of 2: 3 x 2 = 6 - Next is the tens column - I have 3 lots of 3 tens: 3 x 3 = 9 	

	concrete	pictorial	abstract
<p>Column multiplication - with exchange</p> <ul style="list-style-type: none"> - To multiply 2 digit numbers by 1 digit numbers using the formal written methods 	<p>24 x 4 = 96</p> <ul style="list-style-type: none"> - I am multiplying 24, four times. <ul style="list-style-type: none"> - Start by grouping the ones. 4 x 4 is 16, so I need to exchange 10 ones for 1 ten. I have 6 ones left <ul style="list-style-type: none"> - I can now look at the tens column. I have 9 tens, which is 90. 	<p>Base 10</p> <p>24 x 4</p> <ul style="list-style-type: none"> - Begin with the ones column - I have 4 lots of 4. This makes 16, so I need to exchange 10 ones for 1 ten - Now, I can look at the tens column. I have 4 lots of twenty. That is 80. I also have one ten that has been exchanged. That makes 90. 	<p>Column multiplication</p>

Year 5:

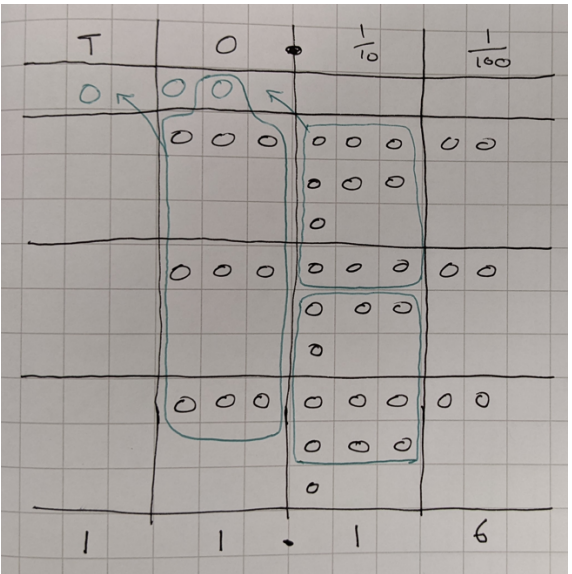
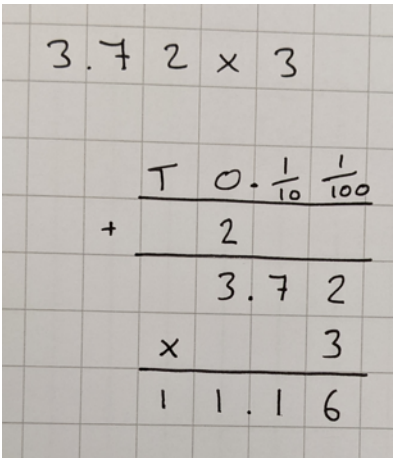
As above, but including the following:

	concrete	pictorial	abstract
To multiply numbers including decimals by 10, 100 and 1000		Reinforce with pictorial from Yrs 3 and 4 if required	 <p>The abstract column contains two sets of handwritten mathematical examples on a grid background. The first set shows the multiplication of 43 by 10 and 100. For 43 x 10, a place value chart with columns labeled T, H, T, O is shown. The number 43 is written in the T and H columns, and 430 is written below it with a bracket indicating multiplication by 10. An arrow points to the right above the number 43. For 43 x 100, the number 43 is written in the T and H columns, and 4300 is written below it with a bracket indicating multiplication by 100. An arrow points to the right above the number 43. The second set shows the multiplication of 4.7 by 10 and 100. A place value chart with columns labeled T, H, T, O and sub-columns for tenths and hundredths is shown. The number 4.7 is written in the T column and the tenths sub-column. For 4.7 x 10, the number 47 is written below it with a bracket indicating multiplication by 10. For 4.7 x 100, the number 470 is written below it with a bracket indicating multiplication by 100.</p>

	concrete	pictorial	abstract
<p>To multiply numbers up to 4 digits by a 1 and 2 digit number</p>		<p>See Y3 and Y4 for pictorial of multiplying by a one digit number. When moving to multiplying by a 2 digit number, use understanding of multiplying by 10 to embed understanding of tens column</p>	<p>316 x 35 Layout and multiplying by ones digit</p>  <p>Multiplying by tens digit</p>  <ul style="list-style-type: none"> - use a different colour pen for tens digit - Underline the zero in the tens digit to help remember adding the placeholder before multiplying

Year 6:

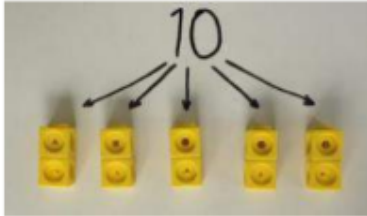
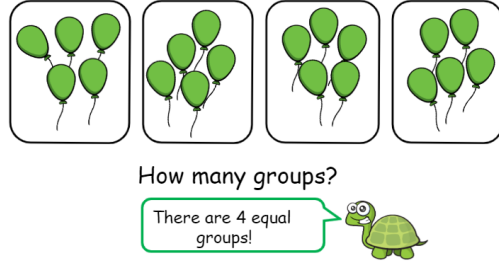
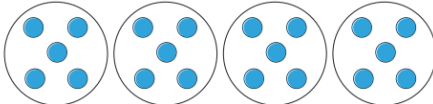
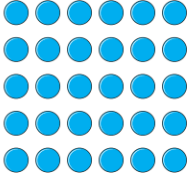
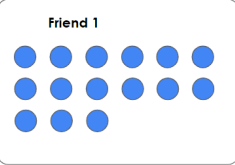
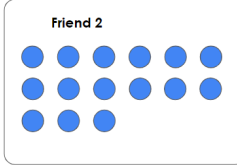
As above, but including the following:

	concrete	pictorial	abstract
To multiply a number with up to 2 decimal places by whole numbers		<p>3.72 x 3</p> 	<p>3.72 x 3</p> 

Calculation guidance: Division


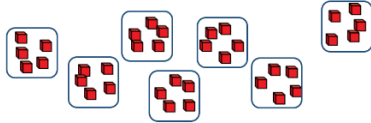
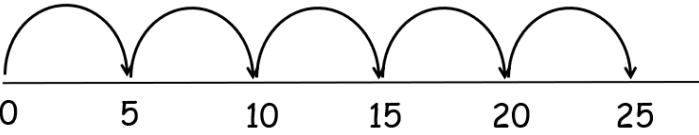
Year 1:

- Focus is on children understanding the concept of division being sharing or grouping into equal groups

	concrete	pictorial	abstract
<p>Making equal groups: grouping</p>	<p>Using objects around the room to put into equal groups</p> <ul style="list-style-type: none"> - How many ___ do I have altogether? - I need to put them into groups of ___ - <i>Make groups, counting out as you go</i> - There are ___ equal groups  <ul style="list-style-type: none"> - I have 10 cubes altogether - I need to put them into groups of 2 - <i>Make groups, counting out in twos</i> - There are 5 equal groups 	<p><i>Drawing out pictures of objects into equal groups</i></p>  <p>How many groups? There are 4 equal groups!</p> <p><i>Moving into using sentence stems to explain the groupings:</i></p> <p>Complete the sentences.</p> <p>a) </p> <p>There are <input type="text"/> counters altogether.</p> <p>There are <input type="text"/> equal groups of <input type="text"/> counters.</p>	
<p>Making equal groups: sharing</p>	<p><i>Difference here is counting out one into each group at a time</i></p>	 <p>Share the counters between 2 friends. How many counters does each friend get?</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <p style="text-align: center; font-size: small;">Friend 1</p>  </div> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <p style="text-align: center; font-size: small;">Friend 2</p>  </div> </div> <p><i>Children draw out counters, one at a time, into each group alternately</i></p>	

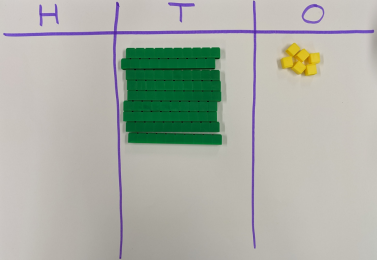
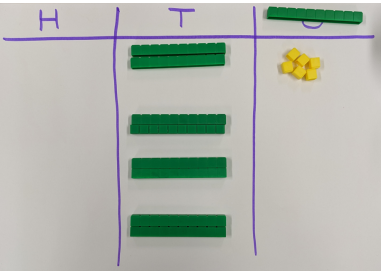
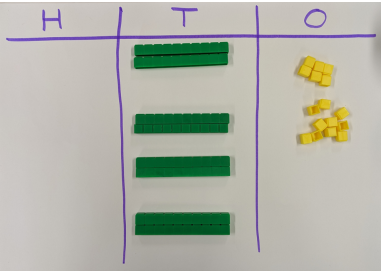

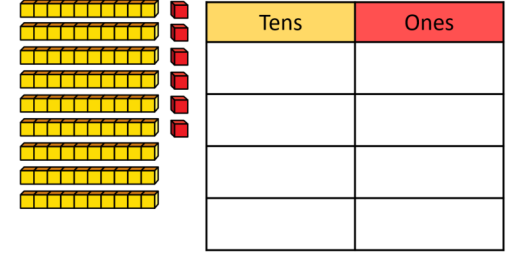
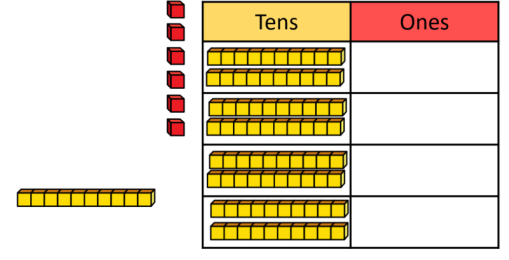
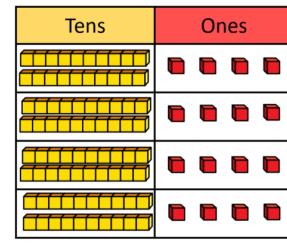
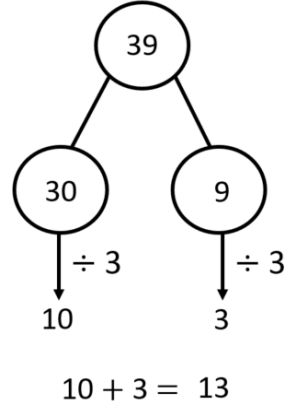
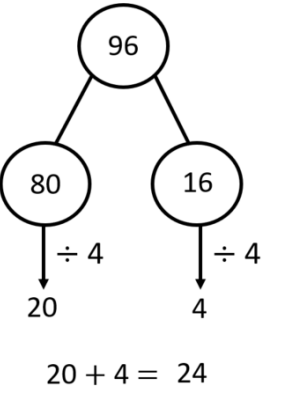
Year 2:

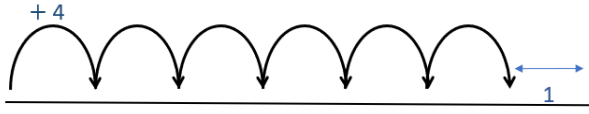























































































- Children should be using their knowledge of times tables that they have been learning to support with understanding of division (and the relationship between the two)
- Continue practising sharing and grouping before moving to numberlines

	concrete	pictorial	abstract
Understand division as dividing into groups of.. (quotative division)	As in Y1	<p>Children should understand the difference between sharing and grouping as strategies, but how they both get to the same answer</p> <p>Sharing </p> <p>There are 7 cubes in each group. $35 \div 5 = 7$</p> <p>Grouping </p> <p>There are 7 groups of 5 $35 \div 5 = 7$</p>	<p>Using concrete / pictorial resources to support with solving number sentence questions</p> <p>a) $6 \div 2$ d) $0 \div 2$ g) $\square \div 2 = 9$ b) $10 \div 2$ e) $\square \div 2 = 5$ h) $\square \div 2 = 11$ c) $14 \div 2$ f) $\square \div 2 = 6$</p>
Using a numberline		 <p>0 5 10 15 20 25</p> <ul style="list-style-type: none"> - I know that $25 \div 5$ can be worked out using my times tables knowledge. - I can count up in 5s until I get to 25. - 5, 10, 15, 20, 25 → I have counted up in 5s five times. - That means there are 5 lots of five in 25. - So, $25 \div 5 = 5$ <p>Could be filled in with children counting the intervals to scaffold</p> <ul style="list-style-type: none"> - The aim is for children to develop an understanding that they can use their times tables to solve division questions 	

Year 3:

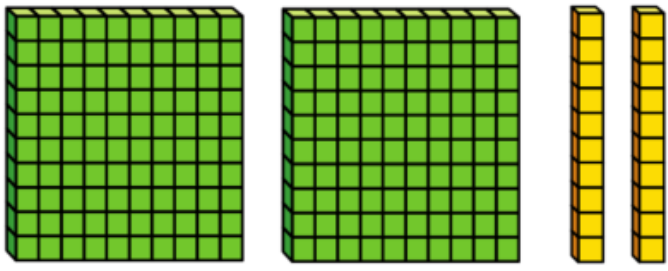
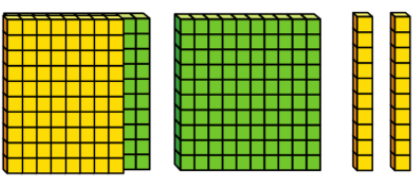
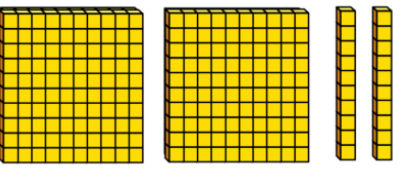
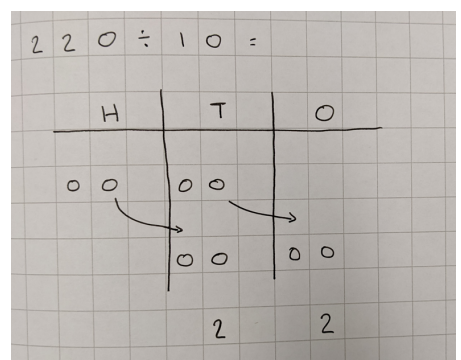
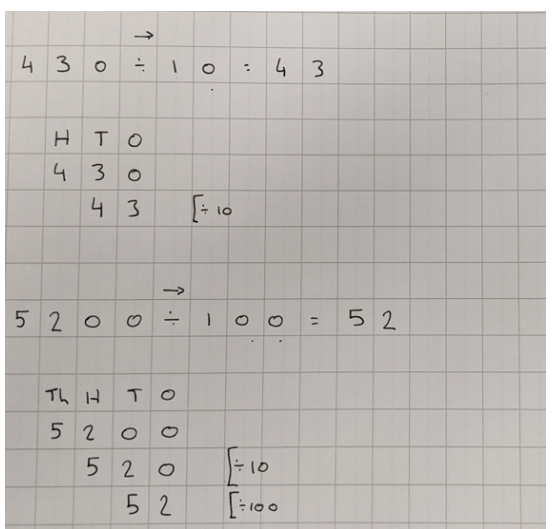
- Continue to embed understanding of sharing and grouping, & repeated addition, before moving on to the following method:

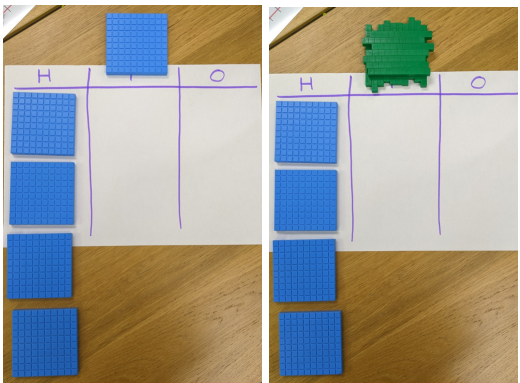
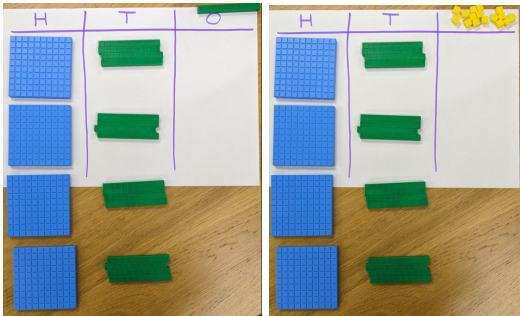
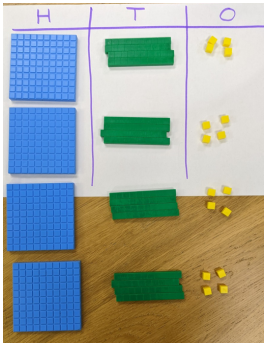
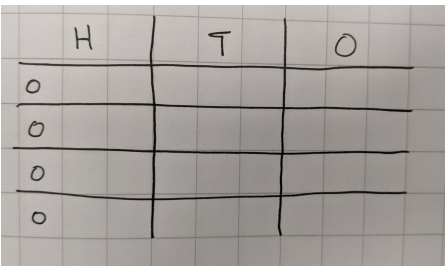
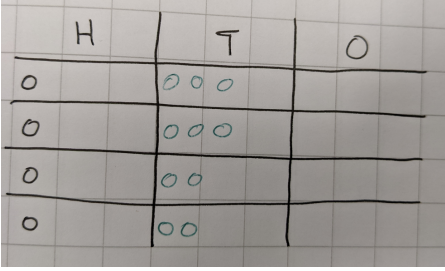
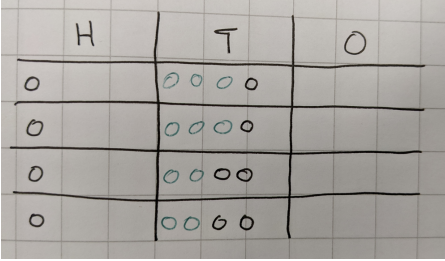
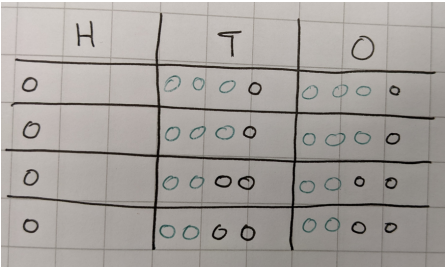
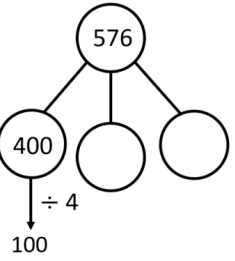
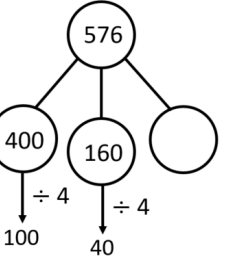
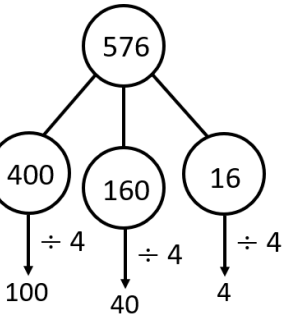
	concrete	pictorial	abstract
To divide 2 digit numbers by 1 digit numbers	<p>96 ÷ 4</p> <ul style="list-style-type: none"> - I am sharing 96, so I need to create the number 9 tens and 6 ones - I am sharing it into 4 equal groups.  <ul style="list-style-type: none"> - I start with the tens column. 9 tens shared into 4 equal groups is 2 in each group with one left over. I need to exchange that left over ten  <ul style="list-style-type: none"> - I now have 16 ones to share into 4 equal groups  <ul style="list-style-type: none"> - 16 ones shared into 4 groups is 4 	<p>96 ÷ 4</p> <ul style="list-style-type: none"> - I am sharing 96 into 4 equal groups. First, I will share my tens  <ul style="list-style-type: none"> - 9 tens shared between four is 2 tens. There is one ten left over, so I need to exchange that for 10 ones  <ul style="list-style-type: none"> - I now have 16 ones. I share these into 4 groups. There are 4 ones in each group.  <ul style="list-style-type: none"> - One group contains 2 tens and 4 ones. That is 24. So, $96 \div 4 = 24$ 	<p>Note: this strategy requires children to have a secure understanding of standard and non-standard partitioning. This strategy lays the foundations for bus stop.</p> <p>39 ÷ 3 =</p> <ul style="list-style-type: none"> - My whole is 39. I am dividing by 3. First, I look at my tens. I know that 3 tens will divide by 3, so I can partition my number as 30 and 9.  <p>96 ÷ 4 =</p> <ul style="list-style-type: none"> - I notice that 9 tens will not divide equally into 4 groups. Therefore, I need to partition my number. I know that 80 divides into 4, so I can partition my number as 80 and 16.  <ul style="list-style-type: none"> - 80 divided into four groups is 20 - 16 divided into four groups is 4 - Therefore, 96 divided by 4 is 24

	concrete	pictorial	abstract																								
<p>To divide 2 digit numbers with remainders</p>		<p>25 ÷ 4</p> <ul style="list-style-type: none"> - I know that dividing by 4 is the same as counting up in 4s until I get to 25.  <p>0 4 8 12 16 20 24 25</p> <ul style="list-style-type: none"> - There are 6 steps of 4. 6×4 is 24. There is one left over. - Therefore, 25 divided by 4 is 6 r1 <p>Then move on to using counters</p> <p>73 ÷ 3</p> <ul style="list-style-type: none"> - I am sharing 73 into 3 equal groups. I start by sharing the tens. <p>$73 \div 3 =$ </p> <table border="1" data-bbox="1365 955 1573 1113"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table> <ul style="list-style-type: none"> - I know that I can share 6 tens equally into 3 groups. However, I have one ten that I can't share. Therefore, I need to exchange 1 ten for 10 ones <p>$73 \div 3 =$ </p> <table border="1" data-bbox="1365 1354 1573 1491"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table> <ul style="list-style-type: none"> - I now have 13 ones. I can share these into three equal groups <p>$73 \div 3 =$ </p> <table border="1" data-bbox="1365 1669 1573 1827"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td>     </td></tr> <tr><td> </td><td>    </td></tr> <tr><td> </td><td>    </td></tr> </tbody> </table>	Tens	Ones							Tens	Ones	 		 		 		Tens	Ones	 	     	 	    	 	    	<p>Stick to pictorial and embed in Y3</p>
Tens	Ones																										
Tens	Ones																										
 																											
 																											
 																											
Tens	Ones																										
 	     																										
 	    																										
 	    																										

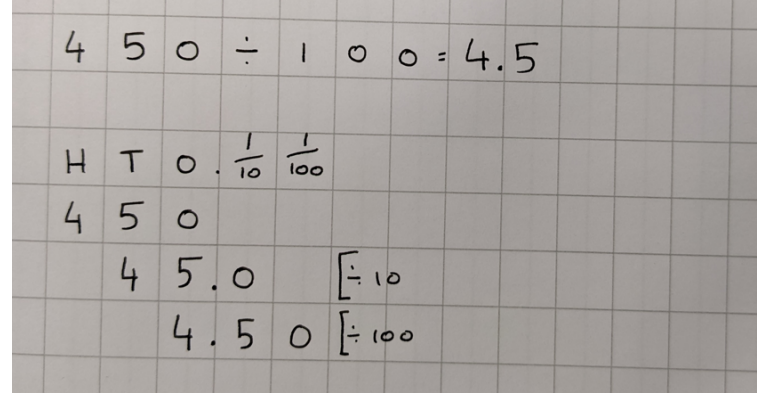
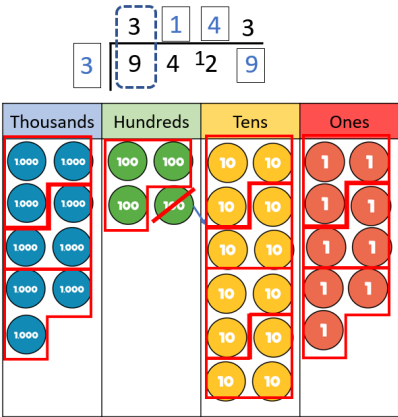
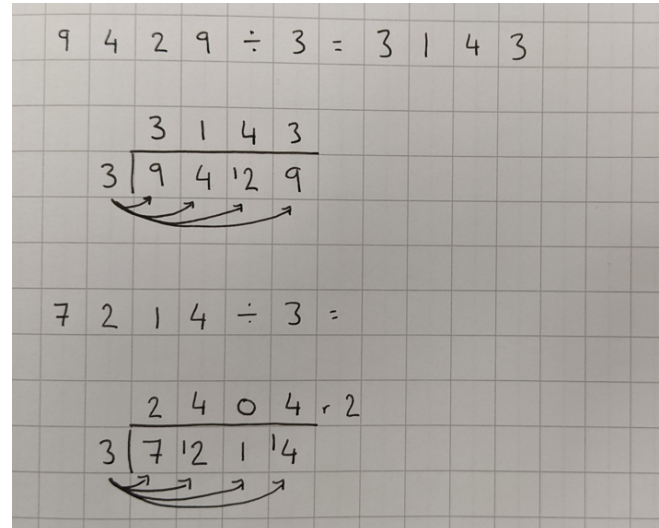
Year 4:

- Children should learn the division facts alongside their times tables knowledge. E.g. If I know that $3 \times 4 = 12$, I also know that $12 \div 4 = 3$

	concrete	pictorial	abstract
To divide whole numbers by 10 and 100 (sticking to whole number quotients)	<p>$220 \div 10 =$</p> <ul style="list-style-type: none"> - We need to see how many groups of ten are in 220  <ul style="list-style-type: none"> - We can count in 10s:   <ul style="list-style-type: none"> - There are 22 groups of 10. Therefore, $220 \div 10$ is 22. - What do children notice? 		

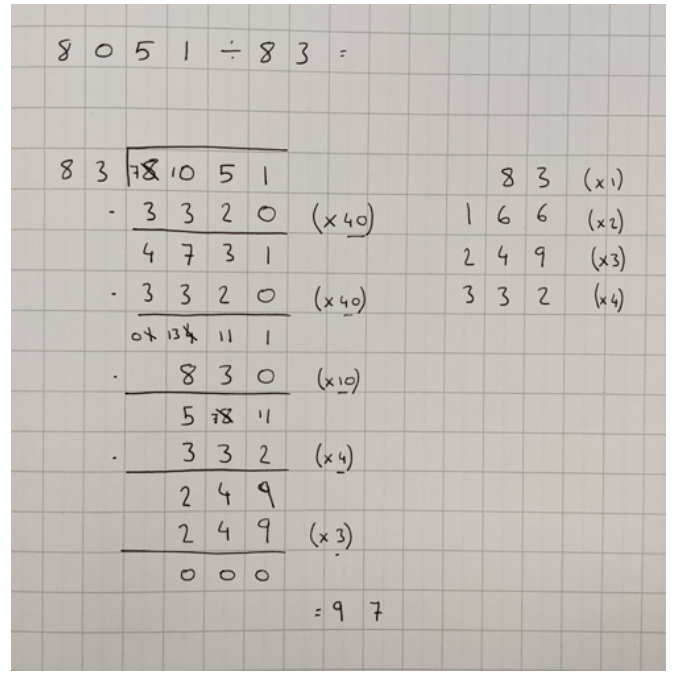
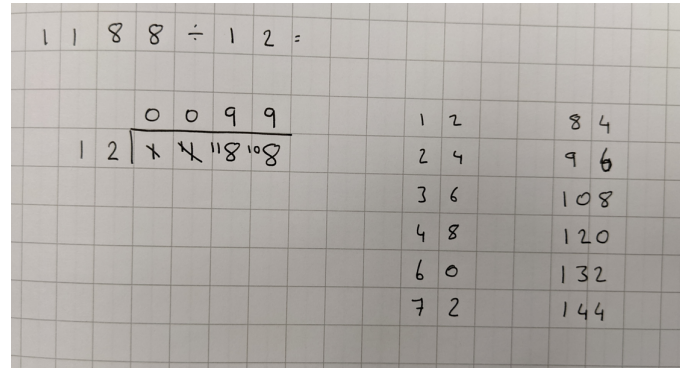
	concrete	pictorial	abstract
Divide 2 digit and 3 digit numbers by 1 digit number	<p>576 ÷ 4</p> <ul style="list-style-type: none"> - First I need to share my 5 hundreds into four equal groups. Each group will have 1 hundred, and there will be 1 hundred left over. I need to exchange this for 10 tens  <ul style="list-style-type: none"> - I now have 17 tens. I need to share those into four equal groups. Each group will have 4 tens, with 1 ten left over. I need to exchange this for 10 ones.  <ul style="list-style-type: none"> - I now need to share 16 ones. When I share this into four equal groups, I have 4 with none left over. - My answer is 144 	<p>576 ÷ 4</p> <ul style="list-style-type: none"> - First I need to share my 5 hundreds into four groups. Each group will have 1 hundred. I will have one hundred left over. I need to exchange that for 10 tens.  <ul style="list-style-type: none"> - Now, I have 17 tens. I need to share them into 4 equal groups. I know that 17 cannot be shared into 4 equal groups, but 16 can.   <ul style="list-style-type: none"> - I need to exchange my 1 ten for 10 ones. I now have 16 ones. When I divide that by 4, each group will have 4 ones.  <ul style="list-style-type: none"> - My answer is 144 	<p>Running alongside the pictorial until children are confident</p> <ul style="list-style-type: none"> - I divide 500 by 4. I know 4 hundreds can be shared into 4 groups. I will have one hundred left over  <ul style="list-style-type: none"> - I now have 17 tens. I know that 16 tens can be divided by 4. I will then have 1 ten left over.  <ul style="list-style-type: none"> - I now have 16 ones. I know this will divide equally by 4. My answer is 144.  <p>When secure, move to numbers where there will be remainders</p>

Year 5:

	concrete	pictorial	abstract
To divide numbers, including decimals, by 10, 100 and 1000	See previous years for embedding	See pictorial from Y4	
To divide numbers up to 4 digits by a 1 digit number	See previous years for embedding		

Year 6:

- As above, but include the following:

	concrete	pictorial	abstract
To divide numbers up to 4 digits by a 2 digit whole number (long division)	See previous years for embedding	See previous years for embedding	 <p>8051 ÷ 83 =</p> <p>83 $\overline{)8051}$</p> <p>- 3320 (x40)</p> <p>4731</p> <p>- 3320 (x40)</p> <p>1411</p> <p>- 830 (x10)</p> <p>581</p> <p>- 332 (x4)</p> <p>249</p> <p>- 249 (x3)</p> <p>000</p> <p>= 97</p> <p>83 (x1) 166 (x2) 249 (x3) 332 (x4)</p> <ul style="list-style-type: none"> - Write out the times tables of the number at the side - Looking to chunk multiples of your divisor out
To divide numbers up to 4 digits by a 2 digit whole number (short division)	See previous years for embedding	See previous years for embedding	 <p>1188 ÷ 12 =</p> <p>12 $\overline{)1188}$</p> <p>0099</p> <p>12 $\overline{)1188}$</p> <p>12</p> <p>24</p> <p>36</p> <p>48</p> <p>60</p> <p>72</p> <p>84</p> <p>96</p> <p>108</p> <p>120</p> <p>132</p> <p>144</p> <ul style="list-style-type: none"> - Write out the times tables of the divisor to help